

# QUANTUM ENTANGLEMENT AND QUANTUM CHROMODYNAMICS

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## **Abstract**

Non-locality or entanglement is an experimentally well established property of quantum mechanics. Here we study the role of quantum entanglement for higher symmetry group like  $SU(3_c)$ , the gauge group of quantum chromodynamics ( QCD ). We show that the hitherto unexplained property of confinement in QCD arises as a fundamental feature of quantum entanglement in  $SU(3_c)$ .

Quantum entanglement, leading to non-locality, is an experimentally well tested aspect of quantum mechanics [1,2,3,4,5,6]. Starting with two particles, now four particle entanglement has been demonstrated [7]. Non-local quantum entanglement forms the basis of the concept of quantum information thereby enabling such phenomena as quantum cryptography [8], dense coding [9], teleportation [9] and quantum computation [10].

The experimental demonstration of quantum entanglement involves polarization states of the photons, spin of the electrons and atoms etc. Hence objects within  $U(1)_{em}$  and representations of the spin  $SU(2)$  group are involved. This entanglement necessarily implies non-locality. Here we would like to study the property of quantum entanglement for representations of higher group, in particular the group  $SU(3)$ . Note that the group  $SU(3_c)$  ( here the subscript c is placed to indicate color degree of freedom ) forms the basis of the theory of strong interaction, the Quantum Chromodynamics, While the electrons and photons are immune to QCD, atoms for whom the quantum entanglement and non-locality has also been demonstrated, are composite systems - consisting of electrons and a nucleus. A nucleus is made up of protons and neutrons whose structure and properties are determined by the rules of QCD. An understanding of non-locality in the quantum atomic systems would involve an understanding of quantum entanglement in QCD. In fact viewed in this manner, it becomes a puzzle as to why quantum non-locality of elementary systems like the electron and the photon is the same as in the composite systems like the atoms. How does the quantum entanglement function in the QCD case to make the above possible? One should also not forget that after all the protons and the neutrons are themselves composite systems of quarks and gluons which in turn are governed by QCD. Hence this understanding may have implications for quantum measurements, quantum cryptography, quantum teleportation and quantum computation.

As a theory of strong interaction QCD is well established both theoretically and experimentally. This is well documented in literature. ( eg. see ref. [11] ). Herein, the protons and neutrons are made up of three constituent quarks and the mesons are made up of a quark-antiquark pair. Note that the quarks belong to the fundamental representation 3 of  $SU(3_c)$ , the gauge group of QCD. In the colour space

$$3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10 \quad (1)$$

$$3 \otimes \bar{3} = 1 \oplus 8 \quad (2)$$

The above coupled with the fact that single coloured quark or any other coloured object like a gluon have not been found in nature, has led to the hypothesis that colour is permanently confined and that colour singlet objects are only found in nature. This so called colour singlet confinement hypothesis is well tested experimentally. Right from the early days of QCD there have been intense efforts to prove the confinement hypothesis in QCD. There have been intriguing hints of confinement in some interpretations, however as of now there has been no definitive proof of confinement in QCD. It is widely expected that confinement arises due to some non-perturbative property of QCD. But, as to what it is, nobody knows. Hence the problem of confinement is still an open one in QCD.

When three quarks or a quark-antiquark come together only colour singlet representations are observed. Hence in the canonical picture it is proposed that in the colour space while the singlet has a finite mass, the octet, decuplet ( and all other higher representations arising in multiquark systems ) are infinitely heavy and hence do not contribute to the low energy spectrum. Also the triplet representation ie. the free quark becomes infinitely heavy when free. Hence the singlets have finite mass and the coloured masses are infinitely heavy [11,12]. Thus it has been a long standing puzzle as to how infinitely heavy free objects can come together in a size of 1 fm to give finite mass bound states.

To appreciate the point further, let us go to the hidden colour concept in the multiquark system. When a neutron and a proton come together to form a bound state of deuteron of approximate size of 2 fm, then at the centre in a region less than a fm or so they should overlap to appear like a six quark bag. As per the colour singlet hypothesis the 6-quark bag looks like,

$$|6q\rangle = \frac{1}{\sqrt{5}}|1\rangle|1\rangle + \frac{2}{\sqrt{5}}|8\rangle|8\rangle \quad (3)$$

where  $|1\rangle$  represents a 3-quark cluster which is singlet in colour space and  $|8\rangle$  represents the same as octet in colour space. Hence  $|8\rangle|8\rangle$  is overall colour singlet. This part is called the hidden colour because as per confinement ideas of QCD these octets cannot be separated out asymptotically and so manifest themselves only within the 6-q colour-singlet system

[13]. Group theoretically the author had earlier obtained the hidden colour components in 9- and 12-quark systems [14,15]. The author found that the hidden colour component of the 9-q system is 97.6% while the 12-q system is 99.8% ie. practically all coloured. These 9- and 12-quark configurations have been found to be relevant in nuclear physics for the  $A=3,4$  nuclei  $^3H$ ,  $^3He$  and  $^4He$ .

Now the problem is that three infinitely heavy quarks come together to form a finite mass colour singlet object while at the same time forming infinite mass octet and decuplet states. When two such colour singlet objects as in eq. 3 come together both the individual singlet as well as octet parts contribute. How come 6-q singlet arises due to two finite mass colour singlets and two infinite mass colour octet objects, both of which contribute similar amounts to the total mass of the 6-q system? Similar problems persist for the 9-q and the 12-q systems [14,15].

Note that what we are doing here is identical to the configuration mixing problem in quantum mechanics - in particular nuclear physics. To determine various properties in nuclear physics one finds that one has to mix several states with a particular quantum number to obtain agreement with the experiments. The configurations which mix to give a particular state are all degenerate or very close in energy. It is always found that configurations which are separated by large energy gaps do not contribute significantly to a particular state. In principle one has an infinite dimensional Hilbert space, however only a finite number of configurations are known to mix to give a state of good quantum number which is physically relevant. Hence in quantum mechanics if any configurations are infinite apart in energy then they are infinitely suppressed in the mixed final state. Therefore it is a puzzle here is as to how the two 3-q states configurations the singlet and the octet, which are infinite energy apart in energy manage to mix in the 6-q system. However this is precisely what confinement means for the 6-q system. Now looking back at eqns 1 and 2 we notice that for confinement in the case of baryons and mesons the same problem of configuration mixing arises. So how does this come about? Thus the confinement problem should be viewed as a fundamental outstanding problem in quantum mechanics.

As shown above, for the ground state hadrons at temperature  $T=0$ , the singlet states have finite energies and the coloured states 3, 8, 10, 27 etc are all expelled to infinite energies. So far there has been no clue as to why it is so in QCD, though there have been several model calculations indicating this

feature [11,12]. Let us now leave the  $T=0$  region and proceed to some finite temperature region and study the problem there. Though it has never been explicitly demonstrated even in a toy model calculation, there is a common feeling that at finite temperatures too the same infinite separation between the singlet and the coloured objects would persist.

Recently we have looked at this specific problem [16]. We looked at the role of higher representations like 8-plet, 27-plet etc. for large hadronic systems like quark stars and objects created in high energy heavy ion collisions.

The orthogonality relation for the associated characters  $\chi_{(p,q)}$  of the  $(p,q)$  multiplet of the group  $SU(3)_c$  with the measure function  $\zeta(\phi, \psi)$  is [16]

$$\int_{SU(3)} d\phi d\psi \zeta(\phi, \psi) \chi_{(p,q)}^*(\phi, \psi) \chi_{(p',q')}(\phi, \psi) = \delta_{pp'} \delta_{qq'} \quad (4)$$

Let us now introduce the generating function  $Z^G$  as

$$Z^G(T, V, \phi, \psi) = \sum_{p,q} \frac{Z_{(p,q)}}{d(p,q)} \chi_{(p,q)}(\phi, \psi) \quad (5)$$

with

$$Z_{(p,q)} = \text{tr}_{(p,q)} \left[ \exp(-\beta \hat{H}_0) \right] \quad (6)$$

$Z_{(p,q)}$  is the canonical partition function. The many-particle-states which belong to a given multiplet  $(p,q)$  are used in the statistical trace with the free hamiltonian  $\hat{H}_0$ ,  $d(p,q)$  is its dimensionality and  $\beta$  is the inverse of the temperature  $T$ . The projected partition function  $Z_{(p,q)}$  can be obtained by using the orthogonality relation for the characters. Hence the projected partition function for any representation  $(p,q)$  is

$$Z_{(p,q)} = d(p,q) \int_{SU(3)_c} d\phi d\psi \zeta(\phi, \psi) \chi_{(p,q)}^*(\phi, \psi) Z^G(T, V, \phi, \psi) \quad (7)$$

Once we have the partition function for any representation  $Z_{(p,q)}$ , then any thermodynamical quantity of interest can be calculated. For example the energy

$$E_{(p,q)} = T^2 \frac{\partial}{\partial T} \ln Z_{(p,q)}. \quad (8)$$

In ref. [16] we projected out different representations like singlet  $(0,0)$ , octet  $(1, 1)$ , 27-plet  $(2, 2)$  etc. on these large hadrons. The most interesting

result we obtained is that for large values of  $TV^{1/3}$  ( where  $V$  is a measure of the size of the composite object ) all representations ; singlet, octet, 27-plet etc are all degenerate in energy ( ie they all have the same energy ) with the unprojected state. There is nothing which favours the colour-singlet representation over the colour-octet at high temperatures. At the same time we also found that our projection technique at low temperatures is able to discriminate between the singlet and the octet states etc by clearly favouring the singlet state over others which are all expelled to infinite energies..

This result is quite general and at high enough temperatures represents the generic property of QCD that all representations have the same energy [16]. Only as the temperature drops do the singlets find themselves favoured over the other colour states and all the coloured states are expelled to infinite energies.

What is our singlet state at finite temperature? It is degenerate in energy with respect to all coloured states. This colour singlet is made from configurations like  $1 \otimes 1$ ,  $8 \otimes 8$ ,  $27 \otimes 27$  etc, that is, all permitted coloured states which can come together to give a singlet. Note that in the said paper [16] we had looked at the chemical potential zero state. We have found that the same holds for non-zero chemical potential as well. Since all these have the same energy, as per the configuration mixing idea in quantum mechanics, at finite temperature there is nothing barring them from contributing to the colour singlet object. How this significant effect at finite temperature manifests itself at  $T=0$  is what we shall discuss below.

As per the standard picture of cosmology the Universe was much hotter at early times. Hence at high enough temperature in the Early Universe as per our calculation in QCD [16], all the coloured states must have existed and were degenerate with the colour singlet states. Being degenerate in energy all kind of configuration mixing giving a particular quantum states were allowed quantum mechanically. Hence, the 6-q state as given by eq. 3 was a quantum mechanically permitted configuration mixed state. In this particular case in the Early Universe clearly the mixing configurations were the singlet and the octet. It was at this stage in the Early Universe that the two states,  $|1\rangle$  with another  $|1\rangle$  and  $|8\rangle$  with another  $|8\rangle$  got entangled quantum mechanically. Similar things happened for all other possible states. Hence a 3-q colour singlet state was an entangled state of three different  $|3\rangle$  coloured states at that temperature. And so on.

As the Universe cooled as shown by us [16], the colour singlet states would

come down in energy while the coloured states would be expelled to infinite energies. And this happens to be the physical situation at present. Hence today when the  $|6 - q\rangle$  state has components  $|1\rangle|1\rangle$  and  $|8\rangle|8\rangle$ , it is because the system remembers that it was entangled in this manner in the Early Universe. It is a manifestation of quantum entanglement of these states in the Early Universe which manifests itself as confinement in QCD at present. We can argue in the same manner for all possible colour singlet states available at present. Hence we propose here that confinement in QCD at present arises as a result of quantum entanglement in  $SU(3_c)$  at finite temperature in the Early Universe.

Note that quantum entanglement in the case of the spin properties of electrons and photon manifests itself as non-locality. In the case of QCD, as shown here, quantum entanglement appears as the local colour confinement. It is quantum correlation which is important. Non-locality or locality depends upon the quantum property under consideration,

Note that the significant effect of the quantum entanglement in QCD leading to confinement is that this is a local effect, meaning that it ensures that QCD entanglement acts only within a distance of a fermi for a nucleon and a few fermis for a nucleus. This ensures that whether you use an electron ( or a photon ) beam, the quantum entanglement effects as studied in the two slit experiments or the EPR kind of experiments, the effect would be the same if you used an atomic beam instead. Had the quantum entanglement effect in QCD been any different than to give confinement in a finite size, the effects for the Universe today would have been unimaginably different! Actually this fact should be seen as experimental confirmation of the ideas presented here.

In summary, we emphasize our recent result in the study of QCD at finite temperature, that all states - coloured as well as singlet are degenerate at finite temperatures [16]. This allows for all kind of quantum configuration mixing giving a particular state in the Early Universe. Hence these configurations get quantum mechanically entangled. As the Universe cooled to reach the present status of finite mass colour singlet states and infinite mass coloured states, the colour singlet states remember as to how they were made up of which coloured entangled states when the Universe was hot. And continues to behave that way at present. Hence confinement in QCD at present, arises as a result of quantum entanglement in QCD at finite temperature in the Early Universe.

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